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Mean-VaR Portfolio Optimization Under ARFIMAX-GARCH Models

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***Abstract* - This paper examined about Mean-VaR portfolio optimization under the models of ARFIMAX-GARCH by using Lagrangean Multiplier approach. Here it is assumed that the stock return and the return of the rupiah exchange rate against the U.S. dollar has non constant mean and volatility, and there is a long memory effects. Where the value the non constant mean are estimated using models of Fractional Autoregressive Integrated Moving Average X (ARFIMAX). While the non constant volatility are estimated using models of Generally Autoregressive Conditional heteroscedasticity (GARCH). In this study, the risk of a portfolio is measured using the Value-at-Risk (VaR). Portfolio optimization analysis is completed form Mean-VaR. The portfolio optimization is performed using the Lagrangean Multiplier and the solution is obtained by the Kuhn-Tucker theorems. As a numerical illustration, the methods mentioned above are used to analyze some stocks that are traded in the capital market in Indonesia.**

***Keywords* - ARFIMAX-GARCH, Value-at-Risk, portfolio optimization, Lagrangean Multiplier, Kuhn-Tucker theorem.**

I. INTRODUCTION

Fluctuations in stock returns often in groups (clusters), where there are groups that have a high volatility time period, followed by high volatility as well in the next time period. Instead, there is a group that has low volatility at a certain period, followed by low volatility as well in the next period. Often there are long term effects or long memory ([1], [3]).

In any investment, investors' attention will be focused on the rate of return investments. Investors will choose promising investment rate of return the highest ([7], [10]). Because it contains elements of the investment uncertainties, investors should consider the risk factors ([4], [6]). The strategy is often used under conditions that risky investment portfolio is formed, which diversified investing in some stocks to reduce the level of risk, and optimize the expected profit rate ([6], [7]).

Value-at-Risk (VaR), until recently it has become popular as a means of measurement of investment risk ([9], [11]). Investment analysis can be done with some models, such as long memory models ([1], [3]), GARCH models ([8]), the Value-at-Risk model ([8]), and portfolio optimization ([12], [13]). Portfolio formation is a popular way to diversify investments ([2], [7]).

In this paper intends to analyze the investment portfolio optimization Mean-VaR, where the stock returns follow the pattern of the time series, assuming that the rate of return of stock has non constant mean and volatility, and there is a long memory effects. The goal is to get the proportions (weights) allocation of funds to be invested in the portfolio formation, so it will be acquired portfolio rate of return expectation maximum and minimum levels of risk.

II. METHODOLOGY

Return. If we let P_{it} is the price of stock i ($i=1,...,N$ with N is the number of stocks that were analyzed) at time t ($t=1,...,T$ with T is the number of observed data), then the stock return r_{it} can be calculated using the formula ([5], [16]):

$$r_{it} = \ln\left(\frac{P_{it}}{P_{it-1}}\right). \qquad (1)$$

The stock return will then be used for modeling the mean model as follows.

Identify the long memory effect. In the next stage we identify the existence of long memory effect in the data return using the rescale range method (R/S) or Geweke and Porter-Hudak (GPH) method. The parameter estimation of fractional difference index d_i ($i=1,...,N$ with N is the number of stocks that were analyzed) is performed using the maximum likelihood method ([8], [15]).

The confidence interval $(1-c)100\%$ for d_i is $\hat{d}_i - z_{(1-\frac{1}{2}\alpha)}\sigma_{d_i} < d_i < \hat{d}_i + z_{(1-\frac{1}{2}\alpha)}\sigma_{d_i}$, where $\hat{d}_i$ denotes estimator of d_i, and z_α denotes the percentile of standard normal distribution at the significance level α. Let μ_{d_i} and σ_{d_i} respectively denote the mean and standard deviation of d_i. We can test the null hypothesis $H_0 : \hat{d}_i = 0$ against $H_1 : \hat{d}_i \neq 0$ using statistic of $z_{d_i} = (d_i - \mu_{d_i})/\sigma_{d_i}$. We reject H_0 if the value of $z_{d_i} < z_{\frac{1}{2}\alpha}$ or $z_{d_i} > z_{1-\frac{1}{2}\alpha}$ ([8], [14]).

Fractional difference process is defined as:

$$(1-B)^{d_i} r_{it} = a_{it}, -0.5 < d_i < 0.5; \qquad (2)$$

where $\{a_{it}\}$ is the error component which is the white noise process, and B denotes the backshift operator, where $B^l r_{it} = r_{it-l}$ ([8]).

ARFIMAX models. One model of time series which can be viewed as an extension of the Fractional Autoregressive Integrated Moving Average (ARFIMA) models is ARFIMAX models, namely ARFIMA models with eXogenous variables. In this model the factors that influence the variable return r_{it} not only the stock return i earlier periods, but also by the other independent variables at a time t. In general, the model of ARFIMAX(p,d,q) can be expressed by the following equation ([8]):

$$(1-B)^{d_i} D(B) r_{it} = \mu_i + C(B)a_{it} + \alpha_1 X_{1t} + ... + \alpha_k X_{kt}, \qquad (3)$$

with B is the backward operator, where $B^l r_{it} = r_{it-l}$.

$D(B) = 1-(a_1 B + a_2 B^2 + ... + a_p B^p)$

$(C(B) = 1 + b_1 B + b_2 B^2 + ... + b_q B^q)$, and μ, $a_1,...,a_p$, $b_1,...,b_q$, $\alpha_1,...,\alpha_q$, real numbers. $\alpha_p, b_q \neq 0$, and $a_{it} \sim IID(0,\sigma_{ai}^2)$. In this model r_{it} and X_{kt} is the time series data and assumed stationary ([8], [14]).

ARFIMAX modeling steps are generally the same as ARFIMA modeling, but in the estimation of the model, the components other independent variables added to the model ([8]). The average modeling process are as follows: (i) identification of the model, ie assign tentative values p and q using correlogram. (ii) The estimated parameter; performed using the least squares method or the maximum likelihood method to estimate the model autoregressive integrated moving average (ARIMA). (iii) diagnostic test; way to test whether the residuals of the mean model is random so it is a relatively small residual, or residual is white noise. (iv) prediction: that using the mean model chosen to predict l - step ahead ([14]).

GARCH models. The non constant volatility of the stock returns is modeled using *generalized autoregressive conditional heteroscedasticity* (GARCH) models. Suppose μ_{it} and σ_{it}^2 respectively denote the non constant mean and volatility of stock return i ($i=1,...,N$ with N is the number of stocks that were analyzed), at the time t ($t=1,...,T$ and T is the period of data observation). The error a_{it} can be calculated as $a_{it} = r_{it} - \mu_{it}$ ([3], [14]). The non constant volatility σ_{it}^2 will follow the GARCH model of degree m and n or GARCH(m,n), if

$$a_{it} = \sigma_{it}\varepsilon_{it},$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{j=1}^{m} \alpha_{ij} a_{it-j}^2 + \sum_{l=1}^{n} \beta_{il}\sigma_{it-l}^2 + \varepsilon_{it} \qquad (4)$$

where α_{i0} is a constant and α_{ij} ($j=1,...,m$) and β_{il} ($l=1,...,n$) denote the parameter coefficients of non constant volatility model of stock return i ($i=1,...,N$). Here we assumed that $\{\varepsilon_{it}\}$ is the sequence independent and identically distribution (iid) random variable with mean zero and variance 1, $\alpha_{i0} > 0$, $\alpha_{ij} > 0$, $\beta_{il} > 0$, and $\sum_{j=1}^{\max(m,n)} (\alpha_{ij} + \beta_{ij}) < 1$ ([8], [14]).

The stages of non constant volatility modeling include: (i) The estimation of mean model, (ii) Testing the effect of ARCH, (iii) Model identification, (iv) Non constant volatility model estimation, (v) Diagnostic test, and (vi) Prediction.

We further use the mean model (3) and the non constant volatility model (4), to calculate $\hat{\mu}_{it} = \hat{r}_{iT}(1)$ and $\hat{\sigma}_{it} = \hat{\sigma}_{iT}(1)$, i.e. the 1-step ahead prediction after time period T of the mean and the variance ([8]).

Portfolio Optimization Model. Let r_p denote the return of portfolio at the time t, and w_i

($i = 1,...,N$) weight of stock i. Return of portfolio r_p can be determined using the equation ([9], [17]):

$$r_p = \sum_{i=1}^{N} w_i r_{it} \text{ ; Terms } \sum_{i=1}^{N} w_i = 1 \text{ and } 0 < w_i < 1 \ (i = 1,...,N). \quad (5)$$

Suppose $\mathbf{\mu}' = (\mu_1,...,\mu_N)$, $i = 1,...,N$ is the mean vector transpose, and $\mathbf{w}' = (w_1,...,w_N)$ the weight vector transpose of portfolio. From equation (9), the weight $\mathbf{w}'$ follows the property $\mathbf{e}'\mathbf{w} = 1$, where $\mathbf{e}' = (1...1)$. The mean of portfolio return μ_p can be estimated using the equation ([7], [16]):

$$\mu_p = \sum_{i=1}^{N} w_i \mu_{it} = \mathbf{w}'\mathbf{\mu}. \quad (6)$$

Suppose given covariance matrix $\mathbf{\Omega} = (\sigma_{ij})_{i,j=1,...,N}$, where $\sigma_{ij} = Cov(r_{it}, r_{jt})$. Variance of the portfolio return σ_p^2 can be expressed as follows:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_{it}^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \text{ ; } i \neq j = \mathbf{w}'\mathbf{\Omega}\mathbf{w}. \quad (7)$$

Where $\sigma_{ij} = Cov(r_{it}, r_{jt})$ denotes the covariance between stock i and stock j ([9], [17]).

Value-at-Risk (*VaR*) of an investment portfolio based on standard normal distribution approach is calculated using the equation [18; 4]:

$$VaR_p = -W_0\{\mathbf{w}'\mathbf{\mu} + z_\alpha(\mathbf{w}'\mathbf{\Omega}\mathbf{w})^{1/2}\}, \quad (8)$$

where W_0 the number of fund is allocated in the portfolio, and z_α is the percentile of standard normal distribution at the significance level α ([4], [17]). When it is assumed $W_0 = 1$ unit, the equation (8) becomes:

$$VaR_p = -\{\mathbf{w}'\mathbf{\mu} + z_\alpha(\mathbf{w}'\mathbf{\Omega}\mathbf{w})^{1/2}\}. \quad (9)$$

A portfolio $\mathbf{w}^*$ *is called* (*Mean-VaR*) *efficient if there is no other portfolio* $\mathbf{w}$ *with* $\mu_p \geq \mu_p^*$ *and* $VaR_p \leq VaR_p^*$ ([9], [13]).

To obtain the efficient portfolio, we used the objective function, to maximize $\{2\tau\mu_p - VaR_p\}$, $\tau \geq 0$ where τ denotes the investor risk tolerance factor. For the investor with the risk tolerance $\tau \geq 0$ therefore we must solve an optimization problem ([9], [13]):

$$Max\{(2\tau + 1)\mathbf{w}'\mathbf{\mu} + z_\alpha(\mathbf{w}'\mathbf{\Omega}\mathbf{w})^{1/2}\} \text{ such that } \mathbf{w}'\mathbf{e} = 1 \quad (10)$$

Since any covariance matrix $\mathbf{\Omega}$ is positive semi-definite, the objective function is quadratic concave. Hence, (10) is a quadratic concave optimization problem. Its Lagrangean function is given by ([18]):

$$L(\mathbf{w}, \lambda) = (2\tau + 1)\mathbf{w}'\mathbf{\mu} + z_\alpha(\mathbf{w}'\mathbf{\Omega}\mathbf{w})^{1/2} + \lambda(\mathbf{w}'\mathbf{e} - 1).$$

Because of the Kuhn-Tucker theorem, the optimality conditions are:

$$\partial L / \partial \mathbf{w} = (2\tau + 1)\mathbf{\mu} + z_\alpha \mathbf{\Omega}\mathbf{w} / (\mathbf{w}'\mathbf{\Omega}\mathbf{w})^{1/2} + \lambda \mathbf{e} = 0 \text{ and } \partial L / \partial \lambda = \mathbf{w}'\mathbf{e} - 1 = 0.$$

For $\tau \geq 0$, we have an optimum portfolio $\mathbf{w}^*$. Based on algebra calculations and $A = \mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e}$, $B = (2\tau + 1)(\mathbf{\mu}'\mathbf{\Omega}^{-1}\mathbf{e} + \mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{\mu})$ and $C = (2\tau + 1)^2 (\mathbf{\mu}'\mathbf{\Omega}^{-1}\mathbf{\mu}) - z_\alpha^2$, we have

$$\lambda = \{-B + (B^2 - 4AC)^{1/2} / 2A \quad (11)$$

by condition $(B^2 - 4AC) \geq 0$, and vector of $\mathbf{w}^*$ is

$$\mathbf{w}^* = \frac{(2\tau + 1)\mathbf{\Omega}^{-1}\mathbf{\mu} + \lambda \mathbf{\Omega}^{-1}\mathbf{e}}{(2\tau + 1)\mathbf{e}'\mathbf{\Omega}^{=1}\mathbf{\mu} + \lambda \mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e}} \quad (12)$$

If the vector of $\mathbf{w}^*$ is substituted into equation (6) and equation (8), then we have the optimum of expected return and Value-at-Risk of portfolios respectively ([18]).

III. Results and Discussion

Data. The data analyzed here include stocks data of TRUB, BMRI, UNTR, BBRI, and HDMT, from January 2007 until April 2010, then successively given the symbol S_1 up to S_5. The data is accessed via the website http://www.finance.go.id//. Stocks data each specified rate of return (or return), and then used for modeling the mean and variance following.

Identify the long memory effects. To identify the effects of long memory, fractional differentiation parameter estimates used d_i ($i = 1,...,5$) in equation (1). To determine the value of the fractional differentiation d_i, used method of Gewek and Porter-Hudak, with the assistance by R software. The result, for the data of stock returns S_1 up to S_5 given in Table-1 below.

Table-1. Identification of Long Memory Effect

Stocks	$\hat{d}_i$	$\hat{\sigma}_{d_i}$	Confidence	z_i	Signi-

			Interval		ficance
S_1	-0.09	0.14	(-0.36, 0.18)	5.85	Yes
S_2	0.07	0.13	(-032, 0.18)	2.48	Yes
S_3	0.34	0.06	(0.22, 0.46)	2.13	Yes
S_4	0.01	0.17	(-0.32, 0.34)	1.61	No
S_5	0.24	0.13	(-0.01, 0.49)	3.54	Yes

To assure the existence of a pattern of long memory, tested the hypothesis $H_0 : \hat{d}_i = 0$ against $H_1 : \hat{d}_i \neq 0$, $i = 1,...,5$. Based on calculations, the statistical z_i, while for the significance level $\alpha = 0.05$, of the standard normal distribution table obtained values $z_{0.05/2} = -1{,}96$ and $z_{(1-0.05/2)} = 1{,}96$. Because the values of z_1, z_2, z_3 and z_5 greater than the value of $z_{(1-0.05/2)}$, concluded that the test results are significant, meaning the data rate of return S_1, S_2, S_3 and S_5 that there are effects of long memory. This is supported also that the confidence interval is in the interval $-0{,}5 < d < 0{,}5$. While the stock returns S_4 there are no long memory effects. The next step uses the data that has fractional differentiation $\hat{d}_i$ to estimate models of the mean and variance.

Modeling of ARFIMAX. The data of stock returns and rupiah exchange rate against USD (r_{it}) will be used to estimate mean model using software of Eviews 6. First, the identification and estimation of mean model. Identification is done through the sample autocorrelation function (ACF) and partial autocorrelation function (PACF). Based on the pattern of ACF and PACF, can be determined the tentative model of each stocks return, as well as through observations by including eXogenous variables will be obtained tentative models. The estimation results indicated that the best models, respectively, that is ARFIMA(5, $\hat{d}_1$,7)-X; ARFIMA(3, $\hat{d}_2$,1)-X; ARFIMA(1, $\hat{d}_3$,3)-X; ARMA(2,2)-X; and ARFIMA(3, $\hat{d}_5$,3)-X. Second, the parameters significance test and models significance test shows that the mean models to all stocks analyzed have been significant. Third, diagnostic tests for these models is done by using the data residual correlogram and Ljung-Box hypotheses test. The test results showed the models residuals are white noise. Results residual normality test a_{it} showed normal distribution. Therefore residual a_{it} for all stocks analyzed have white noise. Equations of the models above will be written together with the following equation GARCH models.

Modeling of GARCH. The first, carried out detection element of autoregressive conditional heteroscedasticity (ARCH) to the residual a_{it}, using the method of ARCH-LM with assistance the software of Eviews 6. The results obtained values of χ^2 (obs * *R-Square*) each stock returns S_1 up to S_5 respectively 132.8219; 52.8483; 9.2341; 17.7126 and 9.32341 each with probability 0.0000 or less than 5%, which means that there are elements of ARCH.

Second, the identification and estimation of variance models. Here use models of generalized autoregressive conditional heterscedasticity (GARCH) refers to equation (4). Based on squared residuals correlogram of a_{it}^2, the ACF and PACF graphs each selected the variance models that may be tentative. Estimation variance models of each the stock returns is done simultaneously with the mean models. The results, obtained the best model are respectively:

- S_1 follow the model of ARFIMA(5, $\hat{d}_1$,7)-X--GARCH(1,1) with a mean equation:
  $r_{1t} = 0.331X_t - 0.397r_{1t-5} + 0.550a_{1t-7} + a_{1t}$ and
  $\sigma_{1t}^2 = 0.001 + 0.073a_{1t-1}^2 + 0.898\sigma_{1t-1}^2 + \varepsilon_{1t}$;
- S_2 follow the model of ARFIMA(3, $\hat{d}_2$,1)-X--GARCH(1,1)-M with a mean equation:
  $r_{2t} = -0.008X_t - 0.933r_{2t-1} + 0.413r_{2t-3}$
  $+ 0.677\sigma_{2t}^2 + 0.0511a_{2t-1} + a_{2t}$ and
  $\sigma_{2t}^2 = 0.002 + 0.055a_{2t-1}^2 + 0.916\sigma_{2t-1}^2 + \varepsilon_{2t}$;
- S_3 follow the model of ARFIMA(1, $\hat{d}_3$,3)-X--GARCH(1,1) with a mean equation:
  $r_{3t} = 0.124 - 0.666r_{3t-1} - 0.492a_{3t-3} + a_{3t}$ and
  $\sigma_{3t}^2 = 0.007 + 0.178a_{3t-1}^2 + 0.488\sigma_{3t-1}^2 + \varepsilon_{3t}$;
- S_4 follow the model of ARMA(2,2)-X--GARCH(1,1)-M with a mean equation:
  $r_{4t} = 0.199X_t + 0.401r_{4t-1} + 0.500r_{4t-2}$
  $+ 0.106\sigma_{4t}^2 - 0.285a_{4t-1} - 0.453a_{4t-2} + a_{4t}$ and
  $\sigma_{4t}^2 = 0.002 + 0.025a_{4t-1}^2 + 0.912\sigma_{4t-1}^2 + \varepsilon_{4t}$; and
- S_5 follow the model of ARFIMA(3, $\hat{d}_5$,3)-X--TGARCH(1,1) with a mean equation:
  $r_{5t} = 0.078X_t - 0.633r_{5t-1} + 0.623r_{5t-3}$
  $+0.689a_{5t-1} - 0.656a_{5t-3} + a_{5t}$ and
  $\sigma_{5t}^2 = 0.003 + 0.025a_{5t-1}^2 + 0.034I_{5t-1} + 0.546\sigma_{5t-1}^2 + \varepsilon_{5t}$

Significance test of the five GARCH models mentioned above produce values of F statistic for S_1 up to S_5 respectively is 5.396421, 5.2566880, 2.886021, 5.779051, and 3.058116, with each having a probability of 0.000000. If the specified significance level of 5%, it is clear that all the probability is less than 5%. So that all of the GARCH models mentioned above have been significant. Based on the ARCH-LM test, residual ε_{it} from the models for stock S_1 up to S_5 was not there ARCH elements, and also has white noise.

Mean and variance models will be used to calculate the values of $\hat{\mu}_{it} = \hat{r}_{it}(1)$ and $\hat{\sigma}_{it}^2 = \hat{\sigma}_{it}^2(1)$ recursively.

Prediction of Mean and Variance. Using the models of mean and variance of the stock returns S_1 up to S_5 mentioned above, hereinafter for calculate the values of $\hat{\mu}_{it} = \hat{r}_{iT}(1)$ and $\hat{\sigma}_{it} = \hat{\sigma}_{iT}(1)$ recursively. The results given in Table-2 below.

Table-2. Mean and Variance

Saham	$\hat{\mu}_{it}$	$\hat{\sigma}_{it}^2$
S_1	0,000213	0,00021
S_2	0,001741	0,00234
S_3	0,000621	0,00036
S_4	0,003072	0,00283
S_5	0,000257	0,00034

The values of mean and variance in Table-2 will be used for the calculation of the investment portfolio optimization as follows.

Portfolio optimization process. The results of the estimated values of mean and variance are given in Table-2 are then used for the calculation of portfolio optimization. Values of the mean of the Table-2 column $\hat{\mu}_{it}$ used to establish mean vector as:

$$\mathbf{\mu}' = (0.000213 \;\; 0.001741 \;\; 0.000621 \;\; 0.003072 \;\; 0.000257).$$

Unit vector established as $\mathbf{e}' = (1\;1\;1\;1\;1)$, that vector whose elements are the numbers 1 as much as the amount of stocks that were analyzed.

Variance values in Table-2 column $\hat{\sigma}_{it}^2$, together with the values of the covariance between stocks $\hat{\sigma}_{ij}$ where $i \neq j$, used to establish the covariance matrix as:

$$\mathbf{\Omega} = \begin{pmatrix} 0.00021 & 0.00003 & 0.00001 & 0.00004 & 0.00002 \\ 0.00003 & 0.00234 & 0.00007 & 0.00008 & 0.00006 \\ 0.00001 & 0.00007 & 0.00036 & 0.00001 & 0.00003 \\ 0.00004 & 0.00008 & 0.00001 & 0.00283 & 0.00002 \\ 0.00002 & 0.00006 & 0.00003 & 0.00002 & 0.00034 \end{pmatrix}$$

and the inverse matrix is:

$$\mathbf{\Omega}^{-1} = 10^3 \begin{pmatrix} 4.8110 & -0.0498 & -0.1004 & -0.0644 & -0.2616 \\ -0.0498 & 0.4323 & -0.0769 & -0.0108 & -0.0660 \\ -0.1004 & -0.0769 & 2.8147 & -0.0047 & -0.2286 \\ -0.0644 & -0.0108 & -0.0047 & 0.3547 & -0.0148 \\ -0.2616 & -0.0660 & -0.2286 & -0.0148 & 2.9892 \end{pmatrix}$$

If the level of significance specified $\alpha = 0.05$, then the percentile values obtained from the standard normal distribution as $z_{0.05} = -1.645$. Percentile values are used to calculate λ using equation (11). The value of the calculations result of this λ is then used to calculate the weight vector $\mathbf{w}^*$ using equation (12). Calculation of weight vector $\mathbf{w}^*$ is done by taking some risk tolerance. The results of the weight vector $\mathbf{w}^*$ then used to calculate mean of the rate of return on the portfolio $\hat{\mu}_p$ using equation (6), and calculate the portfolio risk level VaR_p using equation (9).

For some values of risk tolerance $0 \le \tau < 11.8$ the results of calculations of the portfolio weights are given in Table-3.a, while the calculation of mean rate of return on the portfolio $\hat{\mu}_p$ and the level of portfolio risk VaR_p are given in Table 3.b. If we assume that short sales are not allowed, then for the value of risk tolerance $\tau = 11.8$ is not feasible because it produces negative weights, ie the stock S_1 in the amount of $w_1 =$ **-0.0149**. And so on to the value of risk tolerance $\tau > 11.8$ also not feasible.

Table-3.a. The Results of Portfolio Optimization Process

τ	w_1	w_2	w_3	w_4	w_5
0.0	0.4414	0.0271	0.2521	0.0328	0.2467
0.2	0.4382	0.0285	0.2532	0.0351	0.2451
0.4	0.4349	0.0298	0.2544	0.0374	0.2434
…	…	…	…	…	…
8.6	0.2532	0.1063	0.3196	0.1690	0.1519
8.8	0.2452	0.1096	0.3225	0.1749	0.1478
9.0	0.2367	0.1132	0.3256	0.1810	0.1435
9.2	0.2276	0.1170	0.3288	0.1876	0.1389
9.4	0.2180	0.1211	0.3323	0.1946	0.1341
9.6	0.2076	0.1254	0.3360	0.2021	0.1289
9.8	0.1965	0.1301	0.3400	0.2101	0.1233
10.0	0.1845	0.1352	0.3443	0.2188	0.1172
10.2	0.1714	0.1407	0.3490	0.2283	0.1106
10.4	0.1570	0.1467	0.3542	0.2387	0.1034
10.6	0.1411	0.1534	0.3599	0.2502	0.0954

10.8	0.1234	0.1609	0.3662	0.2631	0.0864
11.0	0.1035	0.1693	0.3734	0.2775	0.0764
11.2	0.0807	0.1789	0.3816	0.2940	0.0649
11.4	0.0542	0.1900	0.3910	0.3132	0.0516
11.6	0.0230	0.2031	0.4023	0.3358	0.0358
11.8	**-0.0149**	0.2191	0.4158	0.3633	0.0167

Table-3.b. The Results of Portfolio Optimization Process

τ	Σw_i	$\hat{\mu}_p$	VaR_p	Max	$\hat{\mu}_p / VaR_p$
0.0	1	0.000462	0.000172	-0.000172	2.686047
0.2	1	0.000471	0.000172	0.000016	2.738372
0.4	1	0.000480	0.000173	0.000211	2.774566
…	…	…	…	…	…
8.6	1	0.000996	0.000236	0.000959	**4.219195**
8.8	1	0.001000	0.000240	0.001360	4.166667
9.0	1	0.001000	0.000245	0.001755	4.081633
9.2	1	0.001100	0.000250	0.002390	4.400000
9.4	1	0.001100	0.000256	0.002824	4.296875
9.6	1	0.001100	0.000262	0.003258	4.198473
9.8	1	0.001200	0.000269	0.004051	**4.460967**
10.0	1	0.001200	0.000276	0.004524	4.347826
10.2	1	0.001200	0.000284	0.004996	4.225352
10.4	1	0.001300	0.000293	0.005947	4.436860
10.6	1	0.001300	0.000304	0.006456	4.276316
10.8	1	0.001400	0.000316	0.007524	4.430380
11.0	1	0.001400	0.000329	0.008071	4.255319
11.2	1	0.001500	0.000345	0.009255	4.347826
11.4	1	0.001600	0.000363	0.010517	4.407713
11.6	1	0.001600	0.000385	**0.011135**	4.155844
11.8	1	*	*	*	*

For each values of risk tolerance $0 \le \tau < 11.8$ produce mean rate of return of the portfolio $\hat{\mu}_p$ and the level of risk VaR_p whose value different. Curved lines between the pair $\hat{\mu}_p$ and VaR_p these establish the efficient frontier. Based on calculations performed previously obtained the results that efficient portfolios lie along the line with the risk tolerance of $0 \le \tau < 11.8$. Where the resulting mean rate of return of the portfolio $\hat{\mu}_p$ minimum of 0.000462 with VaR_p minimum of 0.000172, and mean rate of return of a portfolio of the highest of 0.001600 with VaR maximum of 0.000385. Graphs of efficient frontier like given as Fig.1.

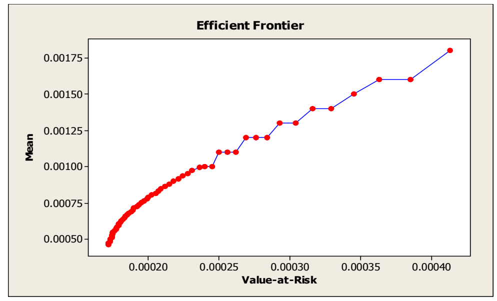

Fig. 1 Efficient Frontier of Investment Portfolio

Among the efficient frontier there is an optimal portfolio and the optimal portfolio is what needs to be sought. Having obtained a set of efficient portfolios, the next step is to determine the composition of the weight of the optimal portfolio. Every investor wants an investment portfolio that can generate mean returns of the portfolios are large, but is expected to be accompanied by a small level of risk. When it is assumed that investor preferences based solely on expected return and risk of the portfolio. The optimal portfolio selection can be done based on the composition of the efficient portfolio weights, that result in the ratio between mean $\hat{\mu}_p$ and the level of risk VaR_p rate of return of the portfolio of the largest. The results of calculating this ratio can be seen in Table-3.b. column $\hat{\mu}_p / VaR_p$ and the graph looks like in Figure-2.

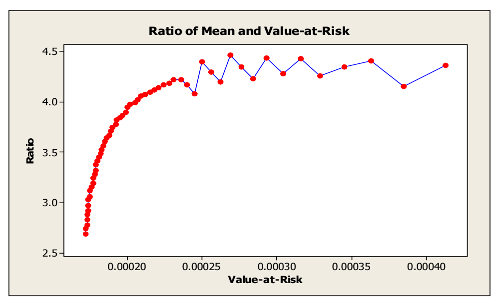

Fig.2. Ratio Between $\hat{\mu}_p$ and VaR_p

The discussion. Based on calculations in Table 3.b., to the values of risk tolerance $0 \le \tau < 11.8$, mean values of $\hat{\mu}_p$ ranging from the smallest of 0.000462 up to the biggest of 0.001600. Risk level of the portfolio, which in this case is measured using VaR_p value ranges from the smallest of 0.000172 up to the biggest of 0.000385. Rate of return and risk usually have a positive relationship, the greater the risks, the greater the rate of return that must be compensated. In Table-3.b., if observed increased levels of portfolio risk VaR_p

also followed by an increase in value of mean rate of return of the portfolio $\hat{\mu}_p$.

Every increase the value of risk tolerance $\tau = 0.0$ up to $\tau = 11.8$ result in an increase in the value of the ratio between mean rate of return on the portfolio $\hat{\mu}_p$ to the level of risk portfolio VaR_p. Increase in the value of risk tolerance $\tau = 0.0$ up to $\tau = 8.6$ accompanied by an increase in the value of the ratio between mean rate of return on the portfolio $\hat{\mu}_p$ to the level of risk of the portfolio VaR_p, but the increase in the value of risk tolerance $\tau = 8.8$ up to $\tau = 9.6$ precisely accompanied by a decline in the value of the ratio between mean rate of return on the portfolio $\hat{\mu}_p$ to level of risk of the portfolio VaR_p. It can be used as one indication that the optimum portfolio value lies in risk tolerance $\tau = 8.6$, which produces mean value of rate of return of the portfolio $\hat{\mu}_p$ = 0.000996 and the level of risk of the portfolio VaR_p = 0.000236. Based on calculations, the optimum portfolio composition resulting weights w_1 = 0.2532, w_2 = 0.1063, w_3 = 0.3196, w_4 = 0.1690, and w_5 = 0.1519. That is, to achieve the optimum portfolio, the beginning capital allocation of one unit, 0.2532 invested in stocks S_1; 0.1063 invested in stocks S_2 and so on.

On the value $\tau = 9.8$, value of the ratio between mean rate of return on the portfolio $\hat{\mu}_p$ to the level of risk of the portfolio VaR_p sizable increases and then decreases. It also shows that the optimum portfolio happens to value of risk tolerance $\tau = 9.8$. However, these events occurred on the efficient frontier with relatively high the level fluctuation. Then, if the assumed short sales are not allowed, then the tolerance for risk $\tau \geq 11.8$ is no longer feasible to invest, particularly in a portfolio consisting of 5 stocks S_1 up to S_5, because it produces negative portfolio weights.

V. CONCLUSIONS

In this paper has been discussed about the Mean-VaR portfolio optimization under models of ARFIMAX-GARCH. Portfolio optimization process of calculations the resulting mean value is equal to 0.000462 minimum portfolio with value of Value-at-Risk of 0.000172. While the mean value of the portfolio is equal to 0.001600 with a maximum value of Value-at-Risk of 0.000385. Along the curved surface that connects the efficient portfolio values of minimum and maximum, there is an optimum portfolio. At these optimum portfolio generated mean value of of 0.000996 with optimum portfolio value of Value-at-Risk of 0.000236. To achieve these optimum portfolio, the composition of the weight allocation of funds invested in stocks till with, successively at 0.2532, 0.1063, 0.3196, 0.1690, and 0.1519. The results this analysis can be used as a reference for investors who will invest in these stocks.

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